Forensic Engineering Determination of the Angle of Lean of a Cycle-Rider System Negotiating a Turn

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Introduction

For a cyclist to negotiate a turn, they must lean the bicycle to the inside of the corner in order to balance the centrifugal force associated with the change in velocity and the gravitational force from gravity. The vector sum of these two forces will be directed between the center of gravity of the rider and the contact patch of the tire on the road surface. Figure 1 shows these forces associated with racing cyclists negotiating a corner.

Figure 1
Riders negotiating a turn

The force on a rider during a turn is dependent upon the radius of the turn, and the velocity of travel. The lean angle of the cyclist is determined by the alignment of the centrifugal force and the rider's weight so as to have the gravitational and inertial forces directed from the center of mass of the rider to the contact patch of the tire on the road. This force balance gives the basic equations for the turning phenomena. The analysis then gives the relationship of the factors of a rider in a turn as a function of speed, radius of curve and the angle of

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lean. The purpose of the analysis is to give the Forensic Engineer the ability to reconstruct the limits of the turning movement of a cyclist in an accident scenario. The factors outlined in this analysis allow the design Civil Engineer to consider the limits of cyclists during a turn. This will enable the designer to safely acclimate bicycle traffic in the design of roads, intersections and bike paths. As in any analysis of this type, the engineer is dealing in ranges and limits, which will also be unique to the individual rider and bicycle.

Engineering Analysis

The Centrifugal acceleration of a rider traveling around a curve of radius R at a velocity V. is:

$$Acceleration = \frac{V^2}{R}$$

The lean angle is determined from the vector relationship, Figure 2, of the centrifugal acceleration force and the gravitational force (weight) of the rider. Acceleration through the turn is limited by the Coefficient of friction between the bicycle tire and riding surface. When the force from the centrifugal acceleration exceeds coefficient of friction of the tire as determined, the maximum cornering capability of the rider and bicycle will be reached for that combination of velocity, lean angle and radius of turn. The frictional force developed by the tire is needed to resist the centrifugal acceleration force.

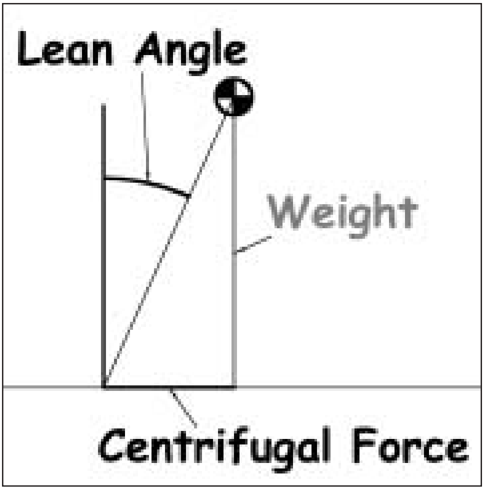

Figure 2
Vector relationship of forces during cornering

$$Force_{acceleration} = \left[\frac{Weight}{g}\right]\frac{V^2}{R}$$

$$Force_{gravity} = Weight$$

$$\tan\theta = \frac{Force_{acceleration}}{Force_{gravity}}$$

$$\tan\theta = \frac{V^2}{gR}$$

Where: V = Velocity
R = curve radius
g = gravitational constant

To illustrate this relationship, an example calculation can be given as follows: Coefficient of friction is a function of the road surface. Typical values can range between 0.3 -0.7. Asphalt has a value of approximately 0.55, but should be determined at each accident site. It is noted that the maximum centrifugal acceleration force is reached when the tire is at the limit of frictional holding force or commonly known as the coefficient of friction. Figure 3 shows the relationships between speed, corner radius and angle of lean for a wide range of riding situations. It must be noted that the high angle of lean shown in Figure 3 is not possible with known surfaces because large lean angles the forces exceed the frictional resistance of the tire road contact surface. If the coefficient of friction between the tire and the road is known, the limiting relationship between speed and cornering radius can be determined.

As an example; Given that a curve radius of 25 feet and a coefficient of friction to be 0.55, the velocity through the curve can be calculated:

$$\frac{V^2}{R} = .55g$$

$$V^2 = (.55)(32.2\,ft/\sec^2)(25\,ft)$$

$$V = \sqrt{(.55)(32.2\,ft/\sec^2)(25\,ft)}$$

$$V = 21\,ft/\sec$$

$$V = 14.3\,mph$$

To determine the angle of lean:

$$\theta = \arctan\left[\frac{V^2}{gR}\right]$$

$$\theta = \arctan\left[\frac{\left(21\frac{ft}{\sec}\right)^2}{32.2\frac{ft}{\sec^2}\,25\,ft}\right]$$

$$\theta = 28.8\deg$$

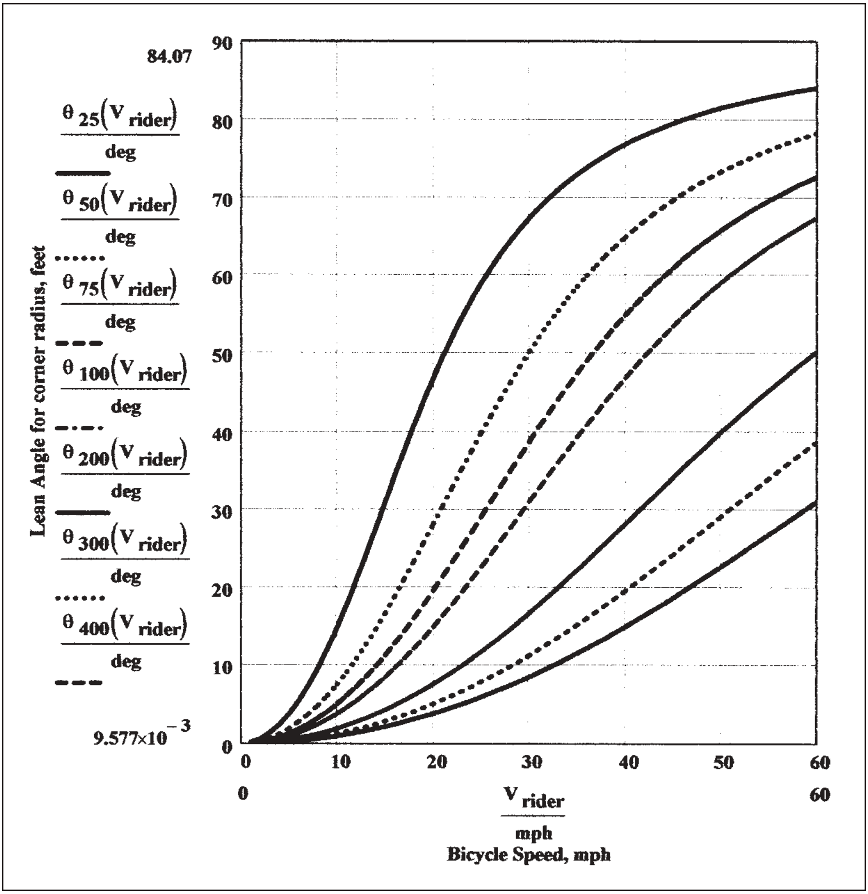

Figure 3
Lean Angle vs. Speed for various corner radiuses

The forensic engineer should determine the radius of the curve at issue and then determine the velocity versus angle of lean as noted in Figure 3.

A cyclist is not capable of angles as large as 65 degrees, but is limited by the ability of the tire to maintain contact with the road surface. The forensic engineer should obtain the original front wheel, or an exemplar of the accident bike front wheel. The wheel should be leaned until the surface bead touches the road. This is the interface between the tire surface and the sidewall. The angle of lean should be obtained and the maximum velocity extrapolated from the angle of lean and velocity curve. In this instance, the mountain bike tire (Figure 4) had an angle of lean of 35°. Therefore, the maximum speed that the cyclist could maintain for this 50 ft. radius curve would be 23 MPH.

It should be noted that road bike tires have an angle of lean less severe than mountain bike tires. An example of an angle of lean of a front road bike tire is noted in Figure 3. In this example, the bead of the tire makes contact with the road at an angle of 32 degrees.

Case Study

In this hypothesized scenario. A motorist struck and killed a cyclist on a 50 ft. radius curve. The motorist had been drinking alcohol all evening before driving. The defense of the motorist in the criminal proceeding was that the cyclist crossed the centerline into the path of the motor vehicle. The motorist maintained that he could have done nothing due to the fact the cyclist came into his path of travel.

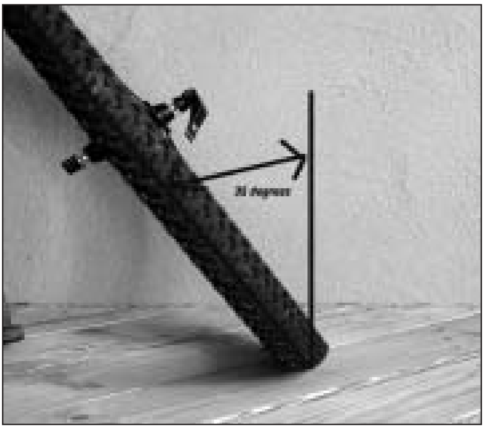

Figure 4
Angle of Lean for a Mountain Bike Tire

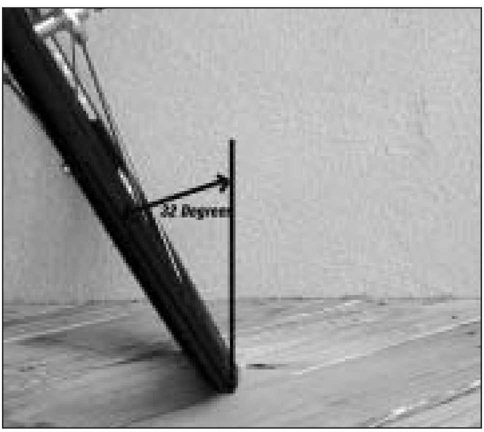

Figure 5
Angle of Lean for a Road Bike Tire

The cyclist was the starting quarterback of the high school football team and was a very accomplished mountain bike racer. It assumed that the cyclist was capable of handling the subject bicycle at the maximum limits of the machine. The maximum angle of lean of the mountain bike tire was 35 degrees. A fact witness who was following the cyclist through the curve testified that the cyclist was traveling at a speed of 20 MPH. An "accident reconstructionist", employed by the criminal defense law firm opined that it was impossible to hold the subject curve at 20 MPH. Therefore, the cyclist would have had to drift over into the oncoming lane of traffic. He also opined that not only was the cyclist drifting into the left hand lane into the path of the driver, but that he was also attempting to make a turn in front of the driver. [The facts of this scenario have been altered for this paper.]

Engineering Analysis

The angle of lean of the front tire at 35 degrees (See Figure 4) gave a maximum speed of approximately 23 MPH. I rode the subject curve on an exemplar bike and found that 20 MPH yielded no sideways float, but as I approached 23 MPH, float to the side occurred. This was consistent with the angle of lean versus velocity curve (Figure 3).

Inspection of the accident scene revealed a pattern of debris several feet prior to the alleged left turn the cyclist was going to take. Unless the motor vehicle driver struck the cyclist, backed up several hundred feet, and then re-hits the cyclist, it was impossible for a left hand turn to have occurred. The facts supported that the cyclist was in the right hand lane. The analysis also revealed that the motor vehicle driver had illegally crossed the centerline and struck the cyclist in the cyclist's lane of travel.

Recommended Procedures for Angle of Lean Analyses

Every accident scene is different. The following procedure has proved to be effective for most angle of lean problems.

1. Survey the subject curve and determine the radius.
2. Obtain an exemplar accident bike and determine the angle of lean.
3. Calculate the angle of lean v velocity for the radius of the curve and plot the curve.
4. Determine the upper speed limit for the cyclist.
5. Ride the accident scene on an exemplar bike at a maximum effort and speed to verify steps 1 through 4.

Conclusion

The angles of lean and velocity relationships developed in this paper are for guidance in determining the upper limit of velocity of a cyclist through a curve. The analysis is not intended to provide an absolute velocity value. There is no substitute for actually obtaining an exemplar bicycle and riding the curve at a maximum physical effort and speed to verify the maximum speed.